ГАЛУЗЕВЕ МАШИНОБУДУВАННЯ

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DETERMINATION OF STRESS-STRAIN STATE OF SHELLS TAKEN INTO ACCOUNT OF STRINGERS AND FRAMES

A methodology for engineering calculations of the strength of cylindrical shells presented, taking into account a frame consisting of a set of stringers and frames. One of the ways to solve this type of problem is to use the method of reduced structural rigidity, when the reinforced shell replaced by a solid one with a rigidity equivalent to the original one. The use of shells of equivalent rigidity in calculation models is justified if it is necessary to evaluate the stress-strain state of objects attached to the shell. The given characteristics of elasticity and rigidity calculated in this work. Equations obtained that describe the stability of cylindrical shells taking into account the frame of stringers and frames. The finite element method used to calculate cylindrical shells supported by stringers and frames. The use of the finite element method makes it possible to calculate the stress-strain state of a mechanical system, taking into account all design features, with full consideration of boundary conditions and specified loads. As an example, the approximation of a tetrahedral finite element shown. The use of the finite element method in solving problems of the strength of reinforced shells leads to a significant increase in the number of finite elements. This leads to too high demands on computing technology in terms of memory capacity and performance. Thus, it is advisable to develop economical and effective methods for calculating the stress-strain state of complex mechanical systems that combine analytical and numerical methods that complement each other and make it possible to evaluate the strength of structures at minimal cost.

Key words: *stress-strain state, shell, stringers, frames, finite element method.*

Formulation of the problem. The tail and side engine compartments of rockets and aircraft fuselages made in the form of frame cylindrical shells. The frame in them is a longitudinal set of stringers and a transverse set of frames. In practice, three types of frames are widely used:

1. Stringers and frames having approximately equal stiffness characteristics, evenly distributed in the longitudinal and transverse directions.
2. Frames have more rigid characteristics than stringers, and they are located much less frequently than stringers.
3. The lightweight frame set of the first type complemented by a set of sparsely spaced reinforced frames and stringers.

Taking into account the complex geometry of structures, an urgent problem arises in constructing an engineering methodology for calculating the strength and stability of frame cylindrical shells, widely used in aircraft construction.

Analysis of recent research and publications. Methods for calculating mechanical systems for stability make it possible quite successfully solve problems for rod and shell structures [1]. The widespread use of reinforced shells in aircraft structures explained by the fact that, with the same mass as smooth shells, they are able to withstand higher levels of compressive stresses. However, the analytical solution to this problem becomes more complex [2]. When using analytical approaches to solve the problem under consideration, one has to face the problem of summing infinite series [2]. Most numerical procedures are based on the finite element solution method [3–7], which is more universal, but requires a justified formation of a finite element model in relation to the problem being solved, therefore, with regard to labor costs, it is important which finite element model is used in the calculations. This work is devoted to the development of analytical and numerical approaches to solving problems of the strength of cylindrical shells and is a development of works [1, 2, 7].

Task statement. The purpose of the article is to determine the main resolving equations and an algorithm for solving the problem of the strength of frame cylindrical shells. There are not enough studies that propose engineering methods for assessing the strength and stability of cylindrical shells taking into account stringers and frames, which allow analytically solving this problem quickly and efficiently in terms of labor costs.

Outline of the main material of the study. Let us consider shells, the reinforcement set of which is located along the lines of the main curvatures. Such shells considered structurally orthotropic. Methods for calculating such shells based on well-known methods for calculating smooth shells. To do this, the reinforced shell replaced by some equivalent smooth shell with different rigidity characteristics along the lines of the main curvatures. After this, the well-developed apparatus of the theory of smooth shells applied to the equivalent shell. In the future, we will consider such shells, the supporting frame of which forms a regular mesh. Let us consider the determination of elastic moduli for a cylindrical shell under the action of tension-compression in the axial direction. When a smooth shell compressed by force P, force does work equal to $A = P\Delta l$, where Δl – is the shortening of the shell.

It is known, that

$$\Delta l = \sum l = \frac{Pl}{E_x F} = \frac{Pl}{2\pi R\delta E_x}.$$

Here E_x – Young's modulus, F – cross-sectional area, R – shell radius, δ – shell wall thickness, l – shell length. Then the work is equal

$$A = \frac{P^2 l}{2\pi R\delta E_x}.$$

When compressed by the force of the reinforced shell, it will be partially absorbed by the stringers and partially by the skin. The total work of these forces is equal to

$$\overline{A} = nP_1\Delta l_1 + P_2\Delta l_2,$$

taking into account that

$$\begin{cases} \Delta l_1 = \varepsilon_1 l = \dfrac{P_1 l}{E_1 F_1}, \\ \Delta l_2 = \varepsilon_2 l = \dfrac{P_2 l}{E_2 F_2}. \end{cases}$$

We have

$$\overline{A} = \left(\frac{nP_1^2}{E_1 F_1} + \frac{P_2^2}{E_2 F_2} \right) l,$$

where n – is the number of stringers, F_1 – is the cross-sectional area of the stringers, F_2 – is the cross-sectional area of the shell, E_1 – is the Young's modulus of the stringer material, E_2 – is the Young's modulus of the shell material.

But also $A = \overline{A}_2$, in addition,

$$\begin{cases} \Delta l_1 = \Delta l_2 = \Delta l, \\ nP_1 + P_2 = P. \end{cases}$$

Taking this into account, we obtain an expression for the reduced Young's modulus E_x of an equivalent smooth shell:

$$E_x = E_2 \left(1 + \frac{nF_1 E_1}{2\pi R\delta E_2} \right).$$

Similarly, we can obtain an expression for Young's modulus E_y:

$$E_y = E_2 \left(1 + \frac{F_3 E_3}{a_2 \delta E_2} \right),$$

where E_3 – is the Young's modulus of the frame material, F_3 – is the cross-sectional area of the frame, and a_2 – is the length of the section between adjacent frames.

For bending rigidity characteristics, the following relations are usually accepted [1, 2]

$$\begin{cases} D_y = \dfrac{E_2 \delta^3}{12(1 - \mu_x \mu_y)} + \dfrac{E_3 I_3}{a_2}, \\ D_x = \dfrac{E_2 \delta^3}{12(1 - \mu_x \mu_y)} + \dfrac{E_1 I_1}{a_1}, \end{cases}$$

where I_1 – is the moment of inertia of the cross-sectional area of the stringer relative to its central axis, parallel to the tangent to the shell circumference, I_3 – is the moment of inertia of the cross-sectional area of the frame relative to its central axis, parallel to the generatrix of the shell, a_1, a_2 – are the distances between adjacent stringers and frames, respectively, μ_x, μ_y – Poisson's ratios. Usually $\mu_x = \mu_y = \mu$, for shear and torsional rigidities, the expressions are taken:

$$D_{кр} = \frac{G_2 \delta^3}{12}, \quad G_2 = \frac{E_2}{2(1+\mu)}.$$

Taking into account the introduced notation, it is possible to express the internal forces in a reinforced shell in the form known in the theory of shells [1, 2]

$$\begin{cases} N_x = \frac{E_x \delta}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y), \\ M_x = -D_x(\lambda_x + \mu\lambda_y), \\ N_y = \frac{E_y \delta}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x), \\ M_y = -D_y(\lambda_y + \mu\lambda_x), \\ N_{xy} = G_2 \varepsilon_{xy}, \\ M_{xy} = -2D_{кр}\lambda_{xy}. \end{cases}$$

For the components of deformations and curvatures we have the usual dependencies

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_1}, \quad \chi_x = -\frac{\partial^2 w}{\partial x^2} - \frac{w}{R_1^2}, \\ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_2}, \quad \chi_y = -\frac{\partial^2 w}{\partial y^2} - \frac{w}{R_2^2}, \\ \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \chi_{xy} = - - \frac{\partial^2 w}{\partial x \partial y}, \end{cases}$$

here u, v – tangential displacements, w – deflection, R_1, R_2 – corresponding radii of curvature.

To obtain differential equations of equilibrium and compatibility of deformations for reinforced shells, the same methods are usually used as for smooth shells. Thus, you can get:

$$\frac{1}{R_1}\frac{\partial^2 \varphi}{\partial y^2} + \frac{1}{R_2}\frac{\partial^2 \varphi}{\partial x^2} + D_x \frac{\partial^4 w}{\partial x^4} + 2(\mu D_y + 2D_x)\frac{\partial^2 w}{\partial y^2 \partial x^2} + D_y \frac{\partial^4 w}{\partial y^4} = q,$$

$$\frac{1}{E_y}\frac{\partial^2 \varphi}{\partial x^2} + \left(\frac{1}{G} - \frac{\mu_x}{E_x} - \frac{\mu_y}{E_y}\right)\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{1}{E_x}\frac{\partial^4 \varphi}{\partial y^4} = \frac{\delta}{R_1}\frac{\partial^2 w}{\partial y^2} + \frac{\delta}{R_2}\frac{\partial^2 w}{\partial x^2},$$

The above formulas use the same notation as in the equations of V.Z. Vlasov for isotropic shells, where φ – is a function of stress and q – is the normal load [1, 2].

In stability problems, the normal load q consists of projections of membrane forces arising in the middle surface of the shell from a given external load and is determined by the formula:

$$q = -N_x^0 \frac{\partial^2 w}{\partial x^2} - N_y^0 \frac{\partial^2 w}{\partial y^2} - 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y},$$

In general terms, the procedure for solving the problem of calculating shells for stability will be reduced to the following. It is necessary to set an expression for the deflection, which would describe as fully as possible the nature of the expected wave formation on the surface of the shell under a given external load. From a geometrical point of view, the expression for deflection should be the equation of the contour of the dents and convexities that form on the surface of the shell due to buckling. On the contour of dents and bulges, the deflection should be zero. If part of the dent contour coincides with the free edge of the shell, then in this case $w \neq 0$, the transverse forces are equal to zero:

$$\begin{cases} \frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y} = 0, \\ \frac{\partial M_y}{\partial y} + 2\frac{\partial M_{xy}}{\partial x} = 0. \end{cases}$$

It should be emphasized that solving the stability problem for reinforced shells with arbitrary boundary conditions requires the use of numerical methods, since a solution can be obtained analytically only for those cases where the solution can be represented as a Fourier series.

To solve a problem using the finite element method, you first need to build a 3D model. An example of 3D modeling is shown in Fig. 1.

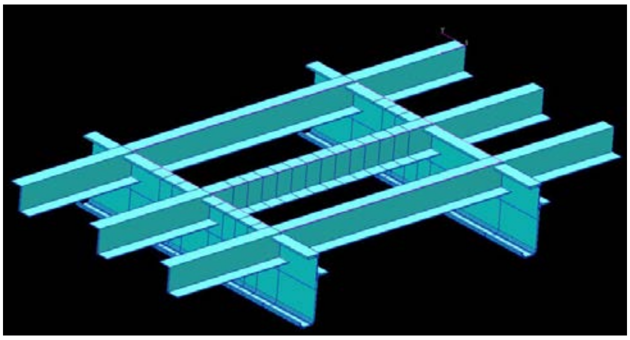

Fig. 1. Frame modeling

Next, the standard FEM procedure is used for calculations [3–6].

Separation of the structure into separate finite elements is a very responsible stage of the calculation. Both the accuracy of the calculation and its laboriousness depend on the correct division. Since this operation has no theoretical basis, its effectiveness depends entirely on the engineering skills of the person engaged in it. Although the use of small elements increases the accuracy of the calculation, it increases the number of unknowns and the order of equations for their determination. In this regard, it is necessary to choose the dimensions of the elements in accordance with the gradients of those values that are determined. In places where the sought value changes quickly, the sizes of the elements are reduced. To build a finite-dimensional model, you can use three-dimensional finite elements in the form of a tetrahedron, within which a linear displacement field is specified:

$$u_x = f_1 + f_2 x + f_3 y + f_4 z,$$

$$u_y = f_5 + f_6 x + f_7 y + f_8 z,$$

$$u_z = f_9 + f_{10} x + f_{11} y + f_{12} z,$$

where $f_1 \ldots f_{12}$ – are arbitrary constants. By equating the nodal points u_x, u_y, u_z to the corresponding nodal displacements, it is possible to express constants in terms of nodal displacements v^e and obtain a dependence in the form of $u = \pm v^e$. Using the usual procedure allows you to find the stiffness matrix of such an element. In the three-dimensional case all six deformation components are taken into account.

The expressions for the stresses inside each element in the general view have the form

$$\sigma^{(m)} = C^{(m)}\varepsilon^{(m)} + \sigma_0^{(m)}$$

where C – is the elasticity matrix of element m, and $\sigma_0^{(m)}$ – is the initial stress in the middle of the element. In a structure consisting of different materials, each element can have its own elasticity matrix.

Conclusions. The paper provides an analytical description of the solution to the problem of stability of cylindrical shells, taking into account stringers and frames. Solving equations are obtained. It is shown that numerical methods, in particular the finite element method, should be used to solve the problems of the stability of rigid cylindrical shells with arbitrary boundary conditions.

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Петрик В.О., Трубачев С.І., Колодежний В.А. ВИЗНАЧЕННЯ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ОБОЛОНОК З УРАХУВАННЯМ СТРИНГЕРІВ І ШПАНГОУТІВ

Представлено методику інженерного розрахунку на міцність циліндричних оболонок з урахуванням каркасу, що складається з набору стрингерів і шпангоутів. Одним із способів розв’язання даного типу завдань є використання методу наведеної жорсткості конструкції, коли підкріплена оболонка замінюється суцільною з жорсткістю, еквівалентною початковій. Використання еквівалентних за жорсткістю оболонок у розрахункових моделях виправдане, якщо необхідно оцінити напружено-деформований стан об’єктів, приєднаних до оболонки. У роботі розраховані наведені характеристики пружності та жорсткості. Отримані рівняння, що описують стійкість циліндричних оболонок з урахуванням каркасу зі стрингерів і шпангоутів. Для розрахунку циліндричних оболонок, підкріплених стрингерами і шпангоутами, застосовується метод скінченних елементів. Застосування методу скінченних елементів дає можливість розрахувати напружено-деформований стан механічної системи з урахуванням усіх конструктивних особливостей, з повним урахуванням граничних умов і заданих навантажень. Як приклад показана апроксимація тетраедричного скінченного елемента. Використання методу скінченних елементів при вирішенні задач міцності підкріплених оболонок призводить до значного зростання кількості скінченних елементів. Це призводить до занадто високих вимог до обчислювальної техніки, з точки зору обсягу пам’яті та швидкодії. Таким чином, доцільно розробляти економічні та ефективні методики розрахунку напружено-деформованого стану складних механічних систем, у яких поєднуються аналітичні та чисельні методи, які доповнюють один одного та дають можливість оцінювати міцність конструкцій з мінімальними витратами.

Ключові слова: *напружено-деформований стан, оболонка, стрингери, шпангоути, метод скінченних елементів.*